FINITE MUTATION CLASSES OF COLOURED QUIVERS

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Abstract. We consider the general notion of coloured quiver mutation and show that the mutation class of a coloured quiver Q, arising from an m-cluster tilting object associated with H, is finite if and only if H is of finite or tame representation type, or it has at most 2 simples. This generalizes a result known for 1-cluster categories.

Introduction

Mutation of skew-symmetric matrices, or equivalently quiver mutation, is very central in the topic of cluster algebras [FZ]. Quiver mutation induces an equivalence relation on the set of quivers. The mutation class of a quiver Q consists of all quivers mutation equivalent to Q. In [BR] it was shown that the mutation class of an acyclic quiver Q is finite if and only if the underlying graph of Q is either Dynkin, extended Dynkin or has at most two vertices.

Cluster categories were defined in [BMRRT] in the general case and in [CCS] in the A_n-case as a categorical model of the combinatorics of cluster algebras. Some cluster categories have a nice geometric description in terms of triangulations of certain polygons, see [CCS, S]. This was used in [To, BTo] to count the number of quivers in the mutation classes of quivers of Dynkin type A and D. In [BRS] they used different methods to count the number of quivers in the mutation classes of quivers of type $\widetilde{A}$.

A generalization of cluster categories, the m-cluster categories, have been investigated by several authors. See for example [BM1, BM2, BT, IY, K, T, W, Z, ZZ]. In [BT] mutation on coloured quivers was defined, and we can define mutation classes of coloured quivers. It is a natural question to ask when the mutation classes of coloured quivers are finite. In this paper we want to show the following theorem, analogous to the main theorem in [BR].

Theorem. *Let k be an algebraically closed field and Q a connected finite quiver without oriented cycles. The following are equivalent for $H = kQ$.*

1. *There are only a finite number of basic m-cluster tilted algebras associated with H, up to isomorphism.*
2. *There are only a finite number of Gabriel quivers occurring for m-cluster tilted algebras associated with H, up to isomorphism.*
3. *H is of finite or tame representation type, or has at most two non-isomorphic simple modules.*
4. *There are only a finite number of τ-orbits of cluster tilting objects associated with H.*
5. *There are only a finite number of coloured quivers occurring for m-cluster tilting objects associated with H, up to isomorphism.*
6. *The mutation class of a coloured quiver Q, arising from an m-cluster tilting object associated with H, is finite.*

1. Background

Let $H = kQ$ be a finite dimensional hereditary algebra over an algebraically closed field k, with Q a quiver with n vertices. The cluster category was defined in [BMRRT] and independently in [CCS] in the A_n case. Consider the bounded derived category $\mathcal{D}^b(H)$ of $\operatorname{mod} H$. Then the cluster category is defined as the orbit category $\mathcal{C}_H = \mathcal{D}^b(H)/\tau^{-1}[1]$, where τ is the Auslander-Reiten translation and [1] is the shift functor.

As a generalization of cluster categories, we can consider the m-cluster categories defined as $\mathcal{C}_H^m = \mathcal{D}^b(H)/\tau^{-1}[m]$. The m-cluster category was shown in [K] to be triangulated. The m-cluster category is a Krull-Schmidt category, an $(m+1)$-Calabi-Yau category, and it has an AR-translate $\tau = [m]$. The indecomposable objects in $\mathcal{C}_H^m$ are of the form $X[i]$, with $0 \leq i < m$, where X is an indecomposable H-module, and of the form $P[m]$, where P is a projective H-module.

An m-cluster tilting object is an object T in $\mathcal{C}_H^m$ with the property that X is in $\operatorname{add} T$ if and only if $\operatorname{Ext}^i_{\mathcal{C}_H^m}(T, X) = 0$ for all $i \in \{1, 2, ..., m\}$. It was shown in [W, ZZ] that an object which is maximal m-rigid, i.e. it has the property that $X \in \operatorname{add} T$ if and only if $\operatorname{Ext}^i_{\mathcal{C}_H^m}(T \oplus X, T \oplus X) = 0$ for all $i \in \{1, 2, ..., m\}$, is also an m-cluster tilting object. They also showed that an m-cluster tilting object T always has n non-isomorphic indecomposable summands.

An almost complete m-cluster tilting object $\bar{T}$ is an object with $n-1$ non-isomorphic indecomposable direct summands such that $\operatorname{Ext}^i_{\mathcal{C}_H^m}(\bar{T}, \bar{T}) = 0$ for $i \in \{1, 2, ..., m\}$. It is known from [W, ZZ] that any almost complete m-cluster tilting object has exactly $m+1$ complements, i.e. there exist $m+1$ non-isomorphic indecomposable objects T' such that $\bar{T} \oplus T'$ is an m-cluster tilting object.

Let $\bar{T}$ be an almost complete m-cluster tilting object and denote by $T_k^{(c)}$, where $c \in \{0, 1, 2, ..., m\}$, the complements of $\bar{T}$. In [IY] it is shown that the complements are connected by $m+1$ exchange triangles

$$T_k^{(c)} \to B_k^{(c)} \to T_k^{(c+1)} \to,$$

where $B_k^{(c)}$ are in $\operatorname{add} \bar{T}$.

An m-cluster tilted algebra is an algebra of the form $\operatorname{End}_{\mathcal{C}_H^m}(T)$, where T is an m-cluster tilting object in $\mathcal{C}_H^m$.

2. Coloured quiver mutation

In the case when $m = 1$ there is a well-known procedure for the exchange of indecomposable direct summands of a cluster-tilting object. Given an almost complete cluster-tilting object, there exist exactly two complements, and the corresponding quivers are given by quiver mutation. For an arbitrary $m \geq 1$, the procedure is a little more complicated. Since an almost complete m-cluster tilting object has, up to isomorphism, exactly $m+1$ complements, the Gabriel quiver does not give enough information to keep track of the exchange procedure. Buan and Thomas therefore defined a class of coloured quivers in [BT], and they define a mutation procedure on such quivers to model the exchange on m-cluster tilting objects. In this section we recall some results from this paper.

To an m-cluster tilting object T, Buan and Thomas associate a coloured quiver Q_T, with arrows of colours chosen from the set $\{0, 1, 2, ..., m\}$. For each indecomposable summand of T there is a vertex in Q_T. If T_i and T_j are two indecomposable summands of T corresponding to vertex i and j in Q_T, there are r arrows from i to j of colour c, where r is the multiplicity of T_j in $B_i^{(c)}$.

They show that such quivers have the following properties.

(1) The quiver has no loops.
(2) If there is an arrow from i to j with colour c, then there exist no arrow from i to j with colour $c' \neq c$.
(3) If there are r arrows from i to j of colour c, then there are r arrows from j to i of colour $m - c$.

They also define coloured quiver mutation, and they give an algorithm for the procedure. Let $Q = Q_T$, for an m-cluster tilting object T, be a coloured quiver and let j be a vertex in Q. The mutation of Q at vertex j is a quiver $\mu_j(Q)$ obtained as follows.

(1) For each pair of arrows

$$i \xrightarrow{(c)} j \xrightarrow{(0)} k$$

where $i \neq k$ and $c \in \{0, 1, ..., m\}$, add an arrow from i to k of colour c and an arrow from k to i of colour $m - c$.
(2) If there exist arrows of different colours from a vertex i to a vertex k, cancel the same number of arrows of each colour until there are only arrows of the same colour from i to k.
(3) Add one to the colour of all arrows that goes into j, and subtract one from the colour of all arrows going out of j.

See Figure 1 for an example.

$$\begin{tikzcd} 1 \arrow[r, bend left, "(0)"] & 2 \arrow[l, bend left, "(2)"] \arrow[r, bend left, "(0)"] & 3 \arrow[l, bend left, "(2)"] \arrow[r, "\mu_1"] & 1 \arrow[r, bend left, "(2)"] & 2 \arrow[l, bend left, "(0)"] \arrow[r, bend left, "(0)"] & 3 \arrow[l, bend left, "(2)"] \arrow[r, "\mu_1"] & 1 \arrow[r, bend left, "(1)"] & 2 \arrow[l, bend left, "(1)"] \arrow[r, bend left, "(0)"] & 3 \arrow[l, bend left, "(2)"] \end{tikzcd}$$

FIGURE 1. Examples of mutation of coloured quivers for Dynkin type A and $m = 2$.

In [BT] the following theorem is proved.

Theorem 2.1. *Let $T = \oplus_{i=1}^{n} T_i$ be an m-cluster tilting object in $\mathcal{C}_H^m$. Let $T' = T/T_j \oplus T_j^{(1)}$ be an m-cluster tilting object where there is an exchange triangle*

$$T_j \to B_j^{(0)} \to T_j^{(1)} \to .$$

Then $Q_{T'} = \mu_j(Q_T)$.

The quiver obtained from Q_T by removing all arrows of colour different from 0 is the Gabriel quiver of the m-cluster tilted algebra $\mathrm{End}_{\mathcal{C}_H^m}(T)$. Quivers of m-cluster tilted algebras can be reached by repeated coloured quiver mutation [ZZ] (see also [BT]).

Proposition 2.2. *Any m-cluster tilting object can be reached from any other m-cluster tilting object via iterated mutation.*

They obtain the following corollary.

Corollary 2.3. *For an m-cluster category $\mathcal{C}_H^m$ of the acyclic quiver Q, all quivers of m-cluster tilted algebras are given by repeated mutation of Q.*

Let us always denote by Q_G the Gabriel quiver of the coloured quiver Q. In this paper we are only interested in coloured quivers which arises from an m-cluster tilting object. Let Q_G be an acyclic quiver and Q the coloured quiver obtained from Q_G by adding the necessary arrows of colour m, i.e. if there exist r arrows from i to j of colour 0, then add r arrows from j to i of colour m. Then the

quivers which arises from m-cluster tilting objects are exactly the quivers mutation equivalent to Q.

Let Q be a coloured quiver with arrows only of colour 0 and m, as above, and where the underlying graph of the Gabriel quiver Q_G is of Dynkin type Δ. Then certainly Q_G is a quiver of an m-cluster tilted algebra. Let us call the set of quivers mutation equivalent to Q the mutation class of type Δ. Certainly, all orientations of Δ (as a Gabriel quiver) is in the mutation class of type Δ.

Figure 2 shows all non-isomorphic coloured quivers in the mutation class of type A_3 for $m = 2$.

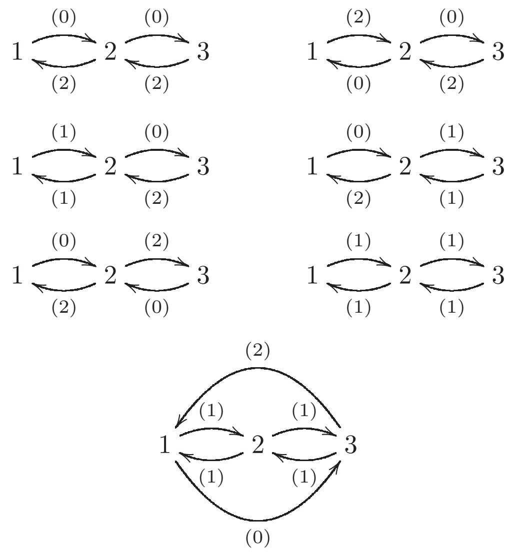

FIGURE 2. All non-isomorphic coloured quivers in the mutation class of A_3 for $m = 2$.

We note that in a mutation class, there can be several non-isomorphic coloured quivers with the same underlying Gabriel quiver, and that the Gabriel quiver of an m-cluster tilted algebra might be disconnected.

To any m-cluster tilting object T there exist a coloured quiver Q_T, but we also have the following.

Lemma 2.4. *Suppose Q is a coloured quiver in some mutation class of a quiver of an m-cluster tilted algebra. Then there exist an m-cluster tilting object T such that $Q = Q_T$.*

Proof. This follows directly from the corollary, since mutation of m-cluster tilting objects corresponds to mutation of coloured quivers. $\square$

We know that $[i]$ is an equivalence on the m-cluster category for all integers i. In particular, $\tau = [m]$ is an equivalence.

Proposition 2.5. *If T is an m-cluster tilting object, then Q_T is isomorphic to $Q_{T[i]}$ for all i*

Proof. It is enough to prove that Q_T is isomorphic to $Q_{T[\pm 1]}$. Suppose there are r arrows in Q_T from i to j with colour c. Let T_i and T_j be the indecomposable direct summands of T corresponding to vertex i and j in Q_T respectively. Let $\bar{T} = T/T_i$

be the almost complete m-cluster tilting object obtained from T by removing T_i. Then there exist an exchange triangle

$$T_i^{(c)} \to B_i^{(c)} \to T_i^{(c+1)} \to$$

with $B_i^{(c)}$ in $\operatorname{add}(\bar{T})$. There are r arrows from i to j, with colour c, so hence T_j has multiplicity r in $B_i^{(c)}$. Clearly $T_i[1]$ and $T_j[1]$ are indecomposable direct summands of $T[1]$ and we have the exchange triangle

$$T_i^{(c)}[1] \to B_i^{(c)}[1] \to T_i^{(c+1)}[1] \to .$$

Since T_j has multiplicity r in $B_i^{(c)}$, $T_j[1]$ has multiplicity r in $B_i^{(c)}[1]$. It follows that there are r arrows in $Q_{T[1]}$ from i to j with colour c. The same proof holds for $[-1]$, and so hence the claim follows. $\square$

3. Finiteness of the number of non-isomorphic m-cluster tilted algebras

In [BR] the authors showed that if Q is a finite quiver with no oriented cycles, then there is only a finite number of quivers in the mutation class of Q if and only if the underlying graph of Q is Dynkin, extended Dynkin or has at most two vertices. In these cases there are only a finite number of non-isomorphic cluster-tilted algebras of some fixed type. In this section we want to prove an analogous result for coloured quivers by generalizing the results and proofs in [BR].

Let $H = kQ$ be a finite dimensional hereditary algebra. We know that H is of finite representation type if and only if the underlying graph of Q is Dynkin. Furthermore, H is tame if and only if the underlying graph of Q is extended Dynkin. Objects in the module category of H, when H is of infinite type, are either preprojective, preinjective or regular. In the case when H is tame, the regular components of the AR-quiver are disjoint tubes of the form $\mathbb{Z}A_\infty / \langle \tau^i \rangle$ for some i, and in the wild case they are of the form $\mathbb{Z}A_\infty$.

If X is a preprojective or preinjective H-module, it is known that X is rigid, i.e. $\operatorname{Ext}^1_H(X, X) = 0$. The following is a well-known result, see for example [R].

Lemma 3.1. *Let $H = kQ$ be a finite dimensional hereditary algebra of infinite representation type, then if H has exactly two simples, no indecomposable regular object is rigid.*

In [W] it was shown that if T is an m-cluster tilting object in $\mathcal{C}_H^m$, then it is induced from a tilting object in $\operatorname{mod} H_0 \vee \operatorname{mod} H_0[1] \vee ... \vee \operatorname{mod} H_0[m-1]$, where H_0 is derived equivalent to H. If H is of finite or tame representation type, it was shown in [BR] that for each indecomposable projective H-module P, there are only a finite number of indecomposable objects X such that $\operatorname{Ext}^1_{\mathcal{C}_H^1}(X, P)$.

Lemma 3.2. *Let $P[i]$ be a shift of an indecomposable projective H-module, where H is of finite or tame representation type. Then there is only a finite set of objects X in $\mathcal{C}_H^m$ with $\operatorname{Ext}^k_{\mathcal{C}_H^m}(X, P[i]) = 0$ for all $k \in \{1, 2, ..., m\}$.*

Proof. We can assume that an m-cluster tilting object is induced from a tilting object in $\operatorname{mod} H \vee \operatorname{mod} H[1] \vee ... \vee \operatorname{mod} H[m-1]$.

It is enough to show that there are a finite number of indecomposable objects X such that $\operatorname{Ext}^1_{\mathcal{C}_H^m}(X, P) = 0$, where P is a projective H-module, since the shift functor is an equivalence on the m-cluster category. It follows from [BR] that there are only a finite number of indecomposable objects X lying inside $\operatorname{mod} H[i]$, with $\operatorname{Ext}^1_{\mathcal{C}_H^m}(X, P[i]) = 0$ for all i.

We have $\mathrm{Ext}^{j+1}_{\mathcal{C}^m_H}(X,P) = \mathrm{Ext}^1_{\mathcal{C}^m_H}(X,P[j])$, so there are only finitely many indecomposable objects X in $\mathrm{mod}\, H[j]$ such that $\mathrm{Ext}^{j+1}_{\mathcal{C}^m_H}(X,P) = 0$. Consequently there are only a finite number of indecomposable objects X such that $\mathrm{Ext}^k_{\mathcal{C}^m_H}(X,P) = 0$ for all $k \in \{1,2,...,m\}$, and we are finished. □

It is known from [BKL] that in the tame case, a collection of one or more tubes is triangulated. We give the proof of the following for the convenience of the reader.

Proposition 3.3. *Let H be a finite dimensional tame hereditary algebra over a field k, and $\mathcal{C}^m_H$ the corresponding m-cluster category. Let*

$$X \to Y \to Z \to$$

be a triangle in $\mathcal{C}^m_H$, where two of the terms are shifts of regular modules. Then all terms are shifts of regular modules.

Proof. It is enough to show that if X and Z are shifts of regular modules, then Y is a shift of a regular module. There exist a homogeneous tube $\mathcal{T}$, i.e. $\tau M = M$ for all $M \in \mathcal{T}$, such that no direct summands of X or Z are in $\mathcal{T}$. Let W be a quasi-simple object in $\mathcal{T}$. We have that W is sincere (see [DR]). We get the exact sequence

$$\mathrm{Hom}(Z,W) \to \mathrm{Hom}(Y,W) \to \mathrm{Hom}(X,W).$$

We have that $\mathrm{Hom}(Z,W) = \mathrm{Hom}(X,W) = 0$, since there are no maps between disjoint tubes. It follows that $\mathrm{Hom}(Y,W) = 0$. Since W is sincere, we have that $\mathrm{Hom}(U,W) \neq 0$ for any projective U, hence for any preprojective since $\tau W = W$. We can do similarly for preinjectives. It follows that all direct summands of Y are shifts of regulars. □

Proposition 3.4. *Let $\mathcal{C}^m_H$ be an m-cluster category, where H is of tame representation type. Let T be an m-cluster tilting object in $\mathcal{C}^m_H$. Then T has, up to τ, at least one direct summand which is a shift of a projective or injective.*

Proof. It is clearly enough to prove that there are no m-cluster tilting objects in $\mathcal{C}^m_H$ with only shifts of regular H-modules as direct summands. So suppose, for a contradiction, that such a T exists.

We can decompose T into indecomposable summands, where $T = T_1 \oplus T_2 \oplus ... \oplus T_n$ and n is the number of simple H-modules. If all direct summands are of the same degree, we already have a contradiction, since a tilting module has at least one direct summand which is preprojective or preinjective (see [R]).

Assume that T_n is a direct summand of degree $k \leq m$. Let $\bar{T} = T_1 \oplus T_2 \oplus ... \oplus T_{n-1}$ be the almost complete m-cluster tilting object obtained from T by removing the direct summand T_n. Then we know that the complements of $\bar{T}$ are connected by $m+1$ AR-triangles,

$$M_{i+1} \to X_i \to M_i \to,$$

where $i \in \{0,1,2,...,m\}$ and $X_i \in \mathrm{add}\, \bar{T}$.

The direct summands of X_i are by assumption shifts of regular modules. We also have that T_n is a shift of a regular module and that it is equal to M_j for some j, since it is a complement of $\bar{T}$. It follows that M_i is a shift of a regular module for all i by Proposition 3.3, since these are connected by the exhange triangles. So all m-cluster tilting objects that can be reached from T by a finite number of mutations, have only regular direct summands.

This leads to a contradiction, because we know from Proposition 2.2 that all m-cluster tilting objects can be reached from T by a finite number of mutations, and a tilting module in H induces an m-cluster tilting object in $\mathcal{C}^m_H$ with at least one direct summand preprojective or preinjective. □

From this it follows that we can assume that an m-cluster tilting object has at least one direct summand which is a shift of a projective up to τ.

We also need a lemma proven in [BR].

Lemma 3.5. *Let H be wild with at least 3 non-isomorphic simples. Let t be a positive integer. Then there is a tilting module T in H with indecomposable direct summands T_1 and T_2, such that $\dim \operatorname{Hom}_H(T_1, T_2) \geq t$.*

To prove the next lemma, which was observed in [BR] for 1-cluster tilted algebras, we use the following fact from [W]. Let $F = \tau^{-1}[m]$. If X and Y are two objects in some chosen fundamental domain in $\mathcal{D}^b(H)$, then $\operatorname{Hom}_{\mathcal{D}^b(H)}(X, F^i Y) = 0$ for all $i \neq 0, 1$.

Lemma 3.6. *If a path in the quiver of an m-cluster tilted algebra goes through two oriented cycles, then it is zero.*

Proof. We have that

$$\operatorname{Hom}_{\mathcal{C}_H}(X, Y) = \oplus_{i \in \mathbb{Z}} \operatorname{Hom}_{\mathcal{D}^b(H)}(X, F^i Y).$$

Let X and Y be two indecomposable m-rigid objects in a chosen fundamental domain. It is well known that since $\operatorname{Ext}_{\mathcal{D}^b(H)}(X, X) = 0$, we have that $\operatorname{End}_{\mathcal{D}^b(H)}(X) = k$. It follows that in an oriented cycle, one of the maps lifts to a map of the form $X \to FY$ in $\mathcal{D}^b(H)$. If there is a path that goes through two oriented cycles, we have a map of the form $X \to FY \to F^2 Z$, and this is 0 by the above. $\square$

The following theorem generalizes the main theorem in [BR].

Theorem 3.7. *Let k be an algebraically closed field and Q a connected finite quiver without oriented cycles. The following are equivalent for $H = kQ$.*

(1) *There are only a finite number of basic m-cluster tilted algebras associated with H, up to isomorphism.*
(2) *There are only a finite number of Gabriel quivers occurring for m-cluster tilted algebras associated with H, up to isomorphism.*
(3) *H is of finite or tame representation type, or has at most two non-isomorphic simple modules.*
(4) *There are only a finite number of τ-orbits of cluster tilting objects associated with H.*
(5) *There are only a finite number of coloured quivers occurring for m-cluster tilting objects associated with H, up to isomorphism.*
(6) *The mutation class of a coloured quiver Q, arising from an m-cluster tilting object associated with H, is finite.*

Proof. (1) implies (2) and (4) implies (5) is clear.

(2) implies (3): Suppose there are only a finite number of quivers occurring for m-cluster tilted algebras associated with H, and let u be the maximal number of arrows between to vertices in the quiver. Then by Lemma 3.6, for any two indecomposable summands T_1 and T_2 of an m-cluster tilting object T, $\dim \operatorname{Hom}_{\mathcal{C}_H^m}(T_1, T_2) < u^{2n}$, where n is the number of simple H-modules. Then it follows from Lemma 3.5 that H is not wild with more than 3 simples.

(3) implies (4): If H is of finite representation type this is clear, since we only have a finite number of indecomposables.

Next, suppose H has at most two non-isomorphic simple modules. If there is only one simple module we have $H \simeq k$, so we can assume there are two simples. Suppose R is a regular indecomposable H-module. Then it follows from Lemma 3.1 that R is not rigid, i.e. $\operatorname{Ext}^1_{\mathcal{C}_H^m}(R, R) \neq 0$. Then we also have that $\operatorname{Ext}^1_{\mathcal{C}_H^m}(R[i], R[i]) \neq 0$ for any $i \in \{1, 2, ..., m-1\}$. Up to τ in $\mathcal{C}_H^m$ we can assume that an m-cluster tilting

object has a direct summand which is a shift of a projective H-module, say $P[j]$. Then $P[j]$ has $m+1$ indecomposable complements. It follows that there are only a finite number of m-cluster tilting objects up to τ, since there are only a finite number of choices for $P[j]$.

Suppose H is tame. By Proposition 3.4, an m-cluster tilting object has at least one direct summand which is a shift of a projective or injective, and hence up to τ we can assume it has an indecomposable direct summand which is a shift of a projective. From Lemma 3.2 we have that there is only a finite number of m-cluster tilting objects with a shift of an indecomposable projective H-module as a direct summand.

(5) implies (6): This is clear, since mutation of m-cluster tilting objects corresponds to mutation of coloured quivers.

We have that (4) implies (1) by using Lemma 2.5. (6) implies (2) is trivial, and so we are done. □

We get the following corollary.

Corollary 3.8. *A coloured quiver Q corresponding to an m-cluster tilting object, has finite mutation class if and only if Q is mutation equivalent to a quiver Q', where Q'_G has underlying graph Dynkin or extended Dynkin, or it has at most two vertices, and there are only arrows of colour 0 and m in Q'.*

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